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The harmonical transmission model: a new approach to multidimensional radiative transfer calculation in gases under consideration of pressure broadening

R. KOCH, S. WITTIG and B. NOLL

Lehrstuhl und Institut für Thermische Strömungsmaschinen, Universität Karlsruhe, Postfach 69 80, D-7500, Karlsruhe 1, Germany

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Abstract—A new method, the harmonical transmission model (HTM), has been developed accounting for the effect of the line structure of gaseous absorption in multidimensional radiative heat transfer calculations. The method is derived from a development of the absorption coefficient and the spectral intensity in Fourier series and a least-square fit over the equation of radiative transfer. For the one-dimensional case, an analytical solution for the spectral mean transmissivity is derived, which can be directly compared to band models. For the multidimensional case, a set of differential equations is presented, which is of the same mathematical form as the equation of radiative transfer. Therefore, any suitable numerical method for the solution of the radiative transfer equation can also be employed to solve the differential equation resulting from the HTM.

1. INTRODUCTION

IN THE last two decades several methods have been developed for the numerical solution of the equation of radiative transfer [1, 2]. When dealing with combustion problems, the equations of fluid mechanics and radiative transfer have to be solved simultaneously. Preferentially those methods are advantageous, where the radiative transfer equation can be written as a set of differential equations of the space coordinates. Among these methods, at present the most established ones are the flux method [3], the moment or spherical harmonics method [4–6] and the discrete ordinate method [7, 8].

For high accuracy the computation of radiative transfer should be carried out on a spectral basis. The difficulty of a spectral computation of the radiative transfer arises from the radiative properties of the gases. Infrared absorption and emission of gases is caused by vibration–rotation bands, which consist of overlapped spectral lines. Because of the line structure of such a vibration–rotation band the absorption coefficient of gases is strongly varying with wavelength. The line structure depends on total pressure as well as on temperature. As the change of the line structure of a vibration–rotation band with total pressure (effect of pressure broadening) has a strong influence on the absorption and emission behaviour of the gas, the line structure must be accounted for in most of the radiative transfer computations of technical combustion systems, which operate at elevated pressures.

In one-dimensional radiative transfer problems, the line structure of such a vibration–rotation band can be accounted for by use of band models [9–11]. All these band models have in common, that a representative spectral mean transmissivity, averaged over a few spectral lines, can be determined. As the transmissivity is an integral term, resulting from an integration of the radiative transfer equation over the optical depth, band models are integral methods. Therefore, in multidimensional radiative transfer problems, band models can only be employed in combination with integral transport methods like the zone method or the Monte-Carlo method, but not in combination with differential transport methods like the flux and the spherical harmonics method, which are usually preferred in combustion modelling.

In this paper, a new method, the harmonical transmission model (HTM), is presented. Combined with differential transport methods for the solution of the radiative transfer equation, the HTM accounts for the effect of the line structure in multidimensional radiative transfer calculations. Since the line structure of a vibration–rotation band is strongly influenced by the total pressure, this method is of particular interest, when the effect of total pressure on the radiative transfer of combustion processes is of importance. A typical application is the theoretical study of radiative heat transfer in gas turbine combustors.

The HTM is based on the same physical principles, that are used in the established band models to describe the line structure of a vibration–rotation band. The basic idea of the HTM is, that the spectral averaging is done directly over the monochromatic radiative transfer equation in contrast to the band models, where the averaging is done over the monochromatic transmissivity.

NOMENCLATURE

d	line spacing
$F_{0\cdots\pm n}$	Fourier coefficients of monochromatic intensity
$\mathbf{F}$	intensity vector
$g_{0\cdots\pm n}$	Fourier coefficients of monochromatic absorption coefficient
G	absorption matrix
H, H^-	transformation matrices
I_λ	monochromatic intensity
$I_{b,\lambda}$	monochromatic intensity of the black body
k_λ	monochromatic absorption coefficient
$\mathbf{n}$	normal vector of the wall
S	line intensity
$\mathbf{s}$	direction vector.

Greek symbols

γ	line half-width
λ	wavelength
τ	transmissivity
$\bar{\tau}$	mean transmissivity
ϕ	dimensionless wavelength parameter
$\mathbf{\Omega}^+$	direction vector of the outgoing intensity at the wall
$\mathbf{\Omega}^-$	direction vector of the incoming intensity at the wall.

Subscript

H.-T. harmonical transmission model.

By the application of the HTM, the resulting differential equations have the same mathematical form as the monochromatic equation of radiative transfer. Therefore, for multidimensional radiative transfer computation, the equations can be solved by using any suitable method for the solution of the radiative transfer equation. However, for the one-dimensional case the set of differential equations derived from the HTM can be solved analytically. As a result, the spectral mean transmissivity is obtained, which can be directly compared to the mean transmissivity of the well-known band models. Consequently, in the one-dimensional case the HTM can be directly compared with band models.

2. THEORETICAL BACKGROUND

If gases are considered, scattering can be neglected and the stationary monochromatic equation of radiative transfer in thermal equilibrium can be written as

$$\mathbf{s}\cdot(\nabla I_\lambda) = -k_\lambda(I_\lambda - I_{b,\lambda}). \tag{1}$$

Because of the line structure of the vibration–rotation bands of infrared-active gases, the spectral absorption coefficient k_λ is strongly varying with the wavelength λ. In addition, k_λ is in general also dependent on the properties of the gas, as well as on the temperature and the partial and total pressure of the gas. A familiar way, used by all band models, to describe the line structure of a vibration–rotation band is to employ the dimensionless parameters S/d and γ/d [12–15].

For a single spectral line the line intensity S is defined as

$$S = \int_\lambda k_\lambda \, \mathrm{d}\lambda. \tag{2}$$

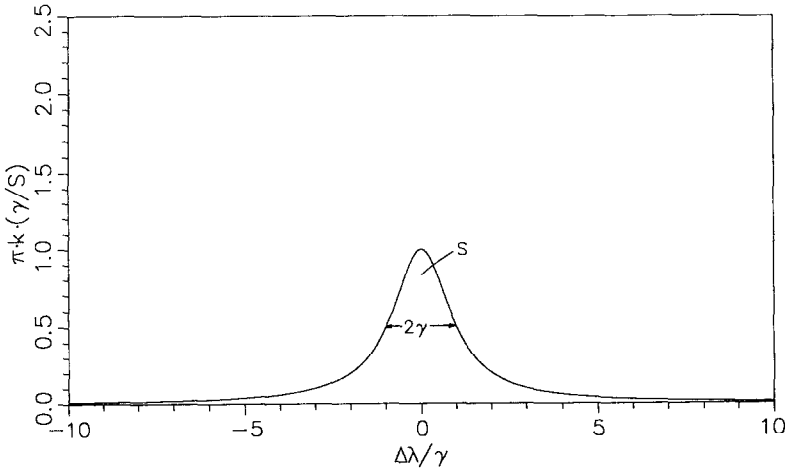

FIG. 1. Definition of the line intensity S and half-width γ.

As shown in Fig. 1, γ represents the half-width at half-height of the spectral line. The vibration–rotation bands of gases consist of an array of spectral lines, which under usual thermodynamic conditions overlap at least partially. Similarly to a single line, an equivalent half-width γ and line spacing d can be defined for an array of a few lines [13]. For the majority of the gases, that give a significant contribution to the radiative heat transfer of combustion processes, the value of

$$\frac{S}{d} = \frac{1}{\Delta\lambda}\int_{\Delta\lambda} k_\lambda \, d\lambda \tag{3}$$

represents the mean absorption coefficient, which the equivalent half-width γ and the equivalent line spacing d can be computed [16, 17] or can be taken from tables [13].

By use of band models a mean transmissivity

$$\bar{\tau} = \frac{1}{\Delta\lambda}\int_{\Delta\lambda} \tau \, d\lambda \tag{4}$$

can be determined. In general, the mean transmissivity $\bar{\tau}$ depends on the quantum mechanical parameters S/d, γ/d and the optical pathlength.

3. ANALYSIS: THE HARMONICAL TRANSMISSION MODEL (HTM)

From a quantum mechanical analysis of the absorption process of two-atomic gases [16], it can be shown that the line structure of a vibration–rotation band consists of nearly equally spaced spectral lines of the same line intensity S and half-width γ. Such a regular structure is also a good approximation for linear, symmetrical three-atomic molecules like carbon dioxide. Therefore, the adequate band model to describe the absorption behaviour of such gases is the regular Elsasser model.

By the Elsasser band model, the structure of a vibration–rotation band is assumed to consist of regularly spaced lines of equal intensity. The lines show a Lorentz profile and do partially overlap. The absorption coefficient of such an Elsasser band is given by (see Fig. 2)

$$k_{\lambda,\mathrm{Elsasser}}\left(\phi = 2\pi\frac{\Delta\lambda}{d}\right) = \frac{S}{d}\cdot\frac{\sinh 2\pi\dfrac{\gamma}{d}}{\cosh 2\pi\dfrac{\gamma}{d} - \dfrac{1}{2}(e^{i\phi}+e^{-i\phi})}. \tag{5}$$

The derivation of the HTM is based on the fact that the absorption coefficient k_λ of a vibration–rotation band shows a periodic structure over a narrow wavelength interval. Because of the periodicity of the line structure, the absorption coefficient k_λ and the spectral intensity I_λ are expanded into complex Fourier series

$$k_\lambda(\phi) = g_0 + g_1\cdot e^{i\phi} + g_{-1}\cdot e^{-i\phi} + \cdots \tag{6}$$

$$I_\lambda(\phi) = F_0 + F_1\cdot e^{i\phi} + F_{-1}\cdot e^{-i\phi} + \cdots \tag{7}$$

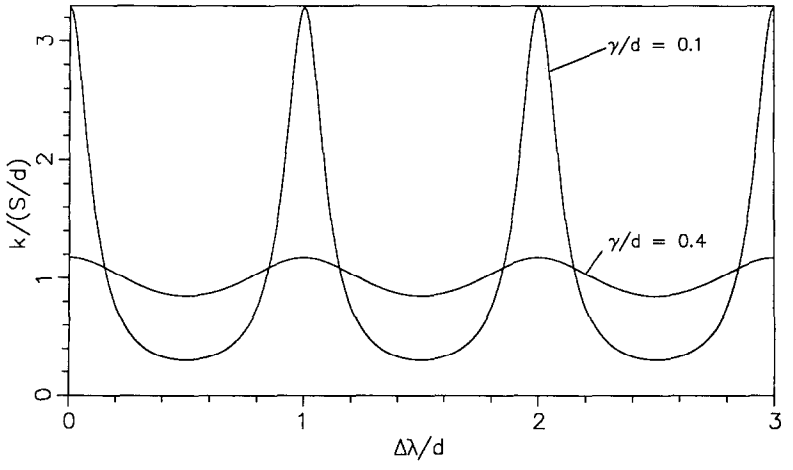

FIG. 2. Line structure of a vibration–rotation band (Elsasser band model).

where

$$\phi = 2\pi \frac{\Delta\lambda}{d}.$$

As the spectral intensity of the black body $I_{b,\lambda}$ is not varying in the narrow wavelength interval $[0 < \Delta\lambda/d < 1]$, the expansion of $I_{b,\lambda}$ in a Fourier series contains only zero-order terms

$$I_{b,\lambda}(\phi) = I_{b,\lambda}. \tag{8}$$

The coefficients g_n of the Fourier expansion of the absorption coefficient (equation (6)) are determined from equation (5) using a least-square fit

$$g_n = \frac{1}{2\pi} \int_{\phi=0}^{2\pi} k_{\lambda,\text{Elsasser}} \cdot e^{-in\phi}\, d\phi. \tag{9}$$

The resulting coefficients are

$$g_0 = \frac{S}{d} \tag{10a}$$

$$g_1 = g_{-1} = \frac{S}{d} \cdot e^{-2\pi(\gamma/d)} \tag{10b}$$

$$g_2 = g_{-2} = \frac{S}{d} \cdot e^{-4\pi(\gamma/d)} \tag{10c}$$

$$g_n = g_{-n} = \frac{S}{d} \cdot e^{-2n\pi(\gamma/d)}. \tag{10d}$$

Because $k_{\lambda,\text{Elsasser}}$ is symmetrical to $\phi = \pi$, it is evident that the imaginary part of the Fourier coefficients g_n vanishes. Therefore, the complex expansion of k_λ can be written as a real one

$$k_\lambda = g_0 + 2g_1 \cdot \cos\phi + 2g_2 \cdot \cos 2\phi + \cdots. \tag{11}$$

In Fig. 3 it is shown how the line structure of an Elsasser band is approximated by the HTM (equation (11)) of first and second order.

The Fourier expansions of the absorption coefficient and the intensity (equations (6) and (7)) are inserted into the monochromatic equation of radiative transfer (equation (1)) and the equation of radiative transfer is averaged over the period interval $[\phi = 0, \phi = 2\pi]$. Because of the orthogonality relation of the Fourier series, averaging in the sense of a least-square fit is achieved by multiplying the equation of radiative transfer by $e^{in\phi}$, where $n = \cdots -2, -1, 0, 1, 2 \cdots$ and integrating from $\phi = 0$ to 2π

$$\int_{\phi=0}^{2\pi} (\text{equation (1)}) \cdot e^{in\phi}\, d\phi. \tag{12}$$

As a result of that quadrature scheme, a coupled system of transfer equations for the complex Fourier

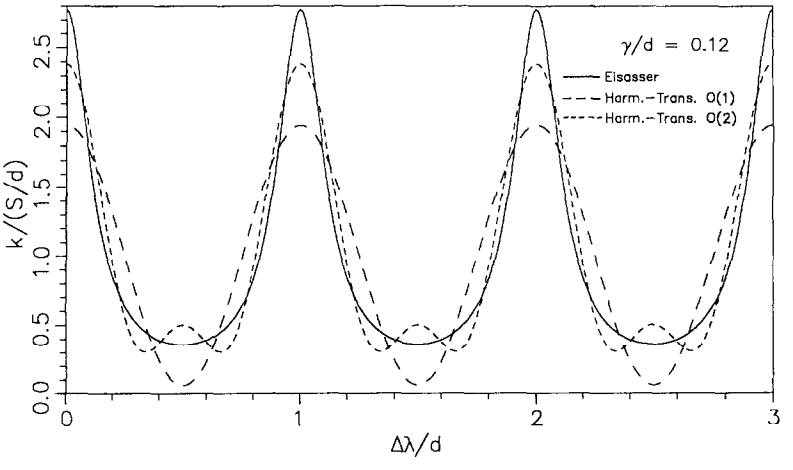

FIG. 3. Line structure as approximated by the HTM for $\gamma/d = 0.12$.

coefficients of the intensity $F_0, F_1, F_{-1}, \ldots$ is obtained

$$\mathbf{s}\cdot\left[\nabla\begin{bmatrix}\vdots\\F_{-2}\\F_{-1}\\F_0\\F_1\\F_2\\\vdots\end{bmatrix}\right]=-\begin{bmatrix}\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\\cdots&g_0&g_{-1}&g_{-2}&\cdots&\cdots&\cdots\\\cdots&g_1&g_0&g_{-1}&g_{-2}&\cdots&\cdots\\\cdots&g_2&g_1&g_0&g_{-1}&g_{-2}&\cdots\\\cdots&\cdots&g_2&g_1&g_0&g_{-1}&\cdots\\\cdots&\cdots&\cdots&g_2&g_1&g_0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\end{bmatrix}\left[\begin{bmatrix}\vdots\\F_{-2}\\F_{-1}\\F_0\\F_1\\F_2\\\vdots\end{bmatrix}-\begin{bmatrix}\vdots\\0\\0\\I_{b,\lambda}\\0\\0\\\vdots\end{bmatrix}\right] \tag{13}$$

Equation (13) can be written in a short form as

$$\mathbf{s}\cdot(\nabla\mathbf{F})=-G(\mathbf{F}-\mathbf{I}_{b,\lambda}). \tag{14}$$

It should be noted that the intensity vector $\mathbf{F}$ as well as the absorption matrix G are dependent on wavelength. The absorption matrix G is complex, hermitian and of the Toeplitz type [18].

4. ONE-DIMENSIONAL RADIATIVE TRANSFER USING THE HARMONICAL TRANSMISSION MODEL

For the one-dimensional case, an analytical solution of equation (13) can be derived. Under the assumption that the incident intensity at $x = 0$ is constant over $\phi = 2\pi(\Delta\lambda/d)$ and that there is no emission of radiation ($I_{b,\lambda} = 0$), an expression for the spectral mean transmissivity can be derived from the HTM. The spectral mean transmissivity can directly be compared to band models.

In order to derive the mean transmissivity from the HTM, the solution of equation (13) in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix}\vdots\\F_{-2}\\F_{-1}\\F_0\\F_1\\F_2\\\vdots\end{bmatrix}=-\begin{bmatrix}\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\\cdots&g_0&g_1&g_2&\cdots&\cdots&\cdots\\\cdots&g_1&g_0&g_1&g_2&\cdots&\cdots\\\cdots&g_2&g_1&g_0&g_1&g_2&\cdots\\\cdots&\cdots&g_2&g_1&g_0&g_1&\cdots\\\cdots&\cdots&\cdots&g_2&g_1&g_0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\end{bmatrix}\begin{bmatrix}\vdots\\F_{-2}\\F_{-1}\\F_0\\F_1\\F_2\\\vdots\end{bmatrix} \tag{15}$$

with g_n given by equations (10a)–(10d) and with the initial conditions

$$F_i=\begin{cases}\bar{I}_0 & \text{for } i=0\\ 0 & \text{for } i\neq 0\end{cases} \tag{16}$$

is required.

If the g_n are chosen according to equations (10a)–(10d), the absorption matrix G becomes symmetrical and real. In this case the system of equations (equation (15)) can be decoupled for the first- and second-order approximation. In this paper only the first-order approximation is considered. For a first-order approximation equation (15) can be decoupled by the transformation

$$\frac{\mathrm{d}}{\mathrm{d}x}H^{-1}\begin{bmatrix}F_{-1}\\F_0\\F_1\end{bmatrix}=-H^{-1}\begin{bmatrix}g_0&g_1&0\\g_1&g_0&g_1\\0&g_1&g_0\end{bmatrix}H\cdot H^{-1}\begin{bmatrix}F_{-1}\\F_0\\F_1\end{bmatrix} \tag{17}$$

where

$$H=\begin{bmatrix}\dfrac{1}{2\sqrt{2}}&\dfrac{-1}{2\sqrt{2}}&\dfrac{-1}{2\sqrt{2}}\\[2mm]\dfrac{1}{2}&0&\dfrac{1}{2}\\[2mm]\dfrac{1}{2\sqrt{2}}&\dfrac{1}{2\sqrt{2}}&\dfrac{-1}{2\sqrt{2}}\end{bmatrix} \tag{18}$$

$$H^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \\ -\sqrt{2} & 0 & \sqrt{2} \\ -\frac{1}{\sqrt{2}} & 1 & -\frac{1}{\sqrt{2}} \end{bmatrix}. \tag{19}$$

The solution of the resulting decoupled system

$$\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ \sqrt{2}(F_1-F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1}) \end{bmatrix} = -\begin{bmatrix} g_0+\sqrt{2}g_1 & 0 & 0 \\ 0 & g_0 & 0 \\ 0 & 0 & g_0-\sqrt{2}g_1 \end{bmatrix}\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ \sqrt{2}(F_1-F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1}) \end{bmatrix} \tag{20}$$

for the initial condition (equation (16)) is

$$F_0 = \tfrac{1}{2}\bar{I}_0 \cdot [\mathrm{e}^{-(g_0+\sqrt{2}g_1)x} + \mathrm{e}^{-(g_0-\sqrt{2}g_1)x}] \tag{21}$$

$$F_1 = F_{-1} = \frac{1}{2}\frac{1}{\sqrt{2}}\bar{I}_0 \cdot [\mathrm{e}^{-(g_0+\sqrt{2}g_1)x} - \mathrm{e}^{-(g_0-\sqrt{2}g_1)x}]. \tag{22}$$

Since F_0 is the spectral averaged value of the intensity, the spectral averaged transmissivity $\bar{\tau}_{\mathrm{H.-T.}}$, which results from the HTM, is defined by

$$\bar{\tau}_{\mathrm{H.-T.}} = \frac{F_0}{\bar{I}_0} = \mathrm{e}^{-g_0 x}\cosh(\sqrt{2g_1}x) \tag{23}$$

or with the use of equations (10a) and (10b)

$$\bar{\tau}_{\mathrm{H.-T.}} = \mathrm{e}^{-(S/d)x}\cosh\left(\sqrt{2\mathrm{e}^{-(2\pi(\gamma/d))}}\frac{S}{d}x\right). \tag{24}$$

This is a very simple formula to describe the spectral averaged transmissivity of a vibration–rotation band with equally spaced lines of the same intensity.

In a similar way also a closed-form expression has been derived for the second-order approximation. Figure 4 illustrates a comparison of the transmission curves of the Elsasser band model and the HTM of zeroth, first and second order. From the comparison of the transmission curves it can be seen that even the HTM of first order gives quite a good approximation to the Elsasser model, and that the HTM of second order is in nearly perfect agreement to the Elsasser model with the added advantage of more accessible computation.

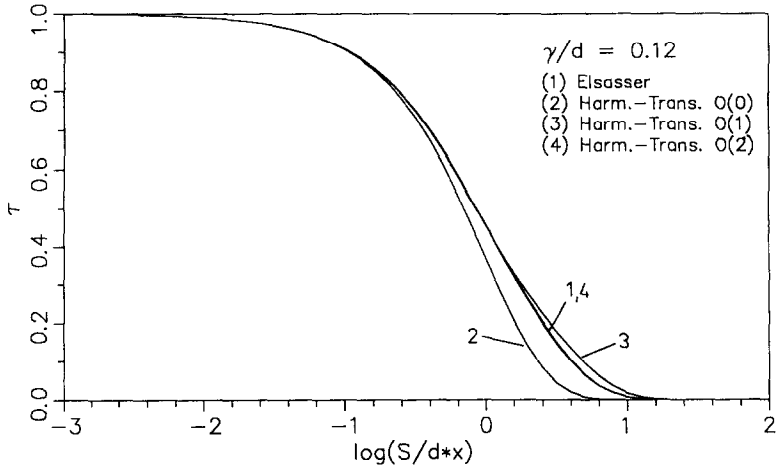

FIG. 4. Comparison of the transmission curves.

5. MULTIDIMENSIONAL RADIATIVE TRANSFER USING THE HARMONICAL TRANSMISSION MODEL

For multidimensional radiative transfer calculation, the system of differential equations (equation (13)) must be solved on the multidimensional domain. The coefficients of the matrix G are taken from equations (10a)–(10d). In a first-order approximation the set of three decoupled differential equations

$$\mathbf{s}\cdot\left(\nabla\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ \sqrt{2}(F_1-F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1})\end{bmatrix}\right)=-\begin{bmatrix} g_0+\sqrt{2}g_1 & 0 & 0 \\ 0 & g_0 & 0 \\ 0 & 0 & g_0-\sqrt{2}g_1\end{bmatrix}\cdot\left(\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ \sqrt{2}(F_1-F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1})\end{bmatrix}-\begin{bmatrix} I_{b,\lambda} \\ 0 \\ I_{b,\lambda}\end{bmatrix}\right) \tag{25}$$

must be solved.

For ideal diffusely reflecting walls, the boundary conditions are

$$F_0(\boldsymbol{\Omega}^+)=(1-\rho)I_{b,\lambda}(\boldsymbol{\Omega}^+)+\frac{\rho}{\pi}\int_{\Omega^-=0}^{2\pi}F_0(\boldsymbol{\Omega}^-)\cdot(\mathbf{n},\boldsymbol{\Omega}^-)\,\mathrm{d}\Omega \tag{26}$$

$$[F_1(\boldsymbol{\Omega}^+)+F_{-1}(\boldsymbol{\Omega}^+)]=\frac{\rho}{\pi}\int_{\Omega^-=0}^{2\pi}[F_1(\boldsymbol{\Omega}^-)+F_{-1}(\boldsymbol{\Omega}^-)]\cdot(\mathbf{n},\boldsymbol{\Omega}^-)\,\mathrm{d}\Omega \tag{27}$$

$$[F_1(\boldsymbol{\Omega}^+)-F_{-1}(\boldsymbol{\Omega}^+)]=\frac{\rho}{\pi}\int_{\Omega^-=0}^{2\pi}[F_1(\boldsymbol{\Omega}^-)-F_{-1}(\boldsymbol{\Omega}^-)]\cdot(\mathbf{n},\boldsymbol{\Omega}^-)\,\mathrm{d}\Omega. \tag{28}$$

The direction vector $\boldsymbol{\Omega}^-$ points to the wall and $\boldsymbol{\Omega}^+$ points away from the wall. The direction of $\mathbf{n}$ is perpendicular to the wall and points away from the wall (see Fig. 5).

With the approximations made here, it can be shown that the second equation of the system

$$\mathbf{s}\cdot(\nabla[\sqrt{2}(F_1-F_{-1})])=-g_0\cdot[\sqrt{2}(F_1-F_{-1})] \tag{29}$$

together with the boundary condition

$$[F_1(\boldsymbol{\Omega}^+)-F_{-1}(\boldsymbol{\Omega}^+)]=\frac{\rho}{\pi}\int_{\Omega^-=0}^{2\pi}[F_1(\boldsymbol{\Omega}^-)-F_{-1}(\boldsymbol{\Omega}^-)]\cdot(\mathbf{n},\boldsymbol{\Omega}^-)\,\mathrm{d}\Omega \tag{30}$$

has the trivial solution

$$(F_1-F_{-1})=0. \tag{31}$$

This fact is easily understandable, when the vanishing sine terms in the Fourier expansion of the absorption coefficient are considered. Therefore, also no sine terms in the Fourier expansion of the intensity should be expected, which is formally expressed by $F_1=F_{-1}$.

Thus it is sufficient to solve the remaining two differential equations

$$\mathbf{s}\cdot\left(\nabla\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1})\end{bmatrix}\right)=-\begin{bmatrix} g_0+\sqrt{2}g_1 & 0 \\ 0 & g_0-\sqrt{2}g_1\end{bmatrix}\cdot\left(\begin{bmatrix} F_0+\frac{1}{\sqrt{2}}(F_1+F_{-1}) \\ F_0-\frac{1}{\sqrt{2}}(F_1+F_{-1})\end{bmatrix}-\begin{bmatrix} I_{b,\lambda} \\ I_{b,\lambda}\end{bmatrix}\right) \tag{32}$$

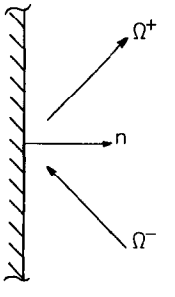

FIG. 5. Definition of $\boldsymbol{\Omega}^+$, $\boldsymbol{\Omega}^-$ and $\mathbf{n}$.

together with the boundary conditions

$$F_0(\mathbf{\Omega}^+) = (1-\rho)I_{b,\lambda}(\mathbf{\Omega}^+) + \frac{\rho}{\pi}\int_{\Omega^- = 0}^{2\pi} F_0(\mathbf{\Omega}^-)\cdot(\mathbf{n},\mathbf{\Omega}^-)\,\mathrm{d}\Omega \tag{33}$$

$$[F_1(\mathbf{\Omega}^+) + F_{-1}(\mathbf{\Omega}^+)] = \frac{\rho}{\pi}\int_{\Omega^- = 0}^{2\pi} [F_1(\mathbf{\Omega}^-) + F_{-1}(\mathbf{\Omega}^-)]\cdot(\mathbf{n},\mathbf{\Omega}^-)\,\mathrm{d}\Omega. \tag{34}$$

If the substitutions

$$A = F_0 + \frac{1}{\sqrt{2}}(F_1 + F_{-1}) \tag{35}$$

$$B = F_0 - \frac{1}{\sqrt{2}}(F_1 + F_{-1}) \tag{36}$$

$$a = g_0 + \sqrt{2}g_1 \tag{37}$$

$$b = g_0 - \sqrt{2}g_1 \tag{38}$$

are introduced, the resulting equations show the same formal structure as the monochromatic equation of radiative transfer in a participating medium (equation (1))

$$\mathbf{s}\cdot(\nabla A) = -a(A - I_{b,\lambda}) \tag{39}$$

$$A(\mathbf{\Omega}^+) = (1-\rho)I_{b,\lambda}(\mathbf{\Omega}^+) + \frac{\rho}{\pi}\int_{\Omega^- = 0}^{2\pi} A(\mathbf{\Omega}^-)\cdot(\mathbf{n},\mathbf{\Omega}^-)\,\mathrm{d}\Omega \tag{40}$$

and

$$\mathbf{s}\cdot(\nabla B) = -b(B - I_{b,\lambda}) \tag{41}$$

$$B(\mathbf{\Omega}^+) = (1-\rho)I_{b,\lambda}(\mathbf{\Omega}^+) + \frac{\rho}{\pi}\int_{\Omega^- = 0}^{2\pi} B(\mathbf{\Omega}^-)\cdot(\mathbf{n},\mathbf{\Omega}^-)\,\mathrm{d}\Omega. \tag{42}$$

Here, the main advantage of the HTM is obvious. The resulting equations are of the same mathematical form as the monochromatic equation of radiative transfer (equation (1)). Therefore, any differential method, which is usually applied to solve the basic equation of radiative transfer (equation (1)), may also be used to solve equations (39) and (41), resulting from the HTM. The only difference arising from the employment of the HTM is, that two equations (one for 'absorption coefficient' a and one for b) have to be solved simultaneously.

From the solutions A, B of these two equations, the line structure of the spectral intensity at every local position can be computed from

$$I(\phi) = \frac{1}{2}(A+B) + \frac{1}{\sqrt{2}}(A-B)\cdot\cos(\phi) \tag{43}$$

where

$$\phi = 2\pi\frac{\Delta\lambda}{d}. \tag{44}$$

6. REGIONS OF VALIDITY

The coefficients g_n of the Fourier expansion of the absorption coefficient k_λ are determined from equations (10a)–(10d). If the line structure parameter γ/d is small and the order of approximation is not chosen high enough, it is possible, that the absorption coefficient might be negative in the interval $[\phi = 0,\ \phi = 2\pi]$, as shown in Fig. 6.

To avoid negative values of k_λ, the order of approximation must be chosen according to Table 1.

As γ/d largely depends on total pressure, the suitability of different orders of approximations depends on total pressure. The regions of total pressure, to which the different orders of approximation are applicable, can be estimated, if a dilute mixture of the infrared-active gas in nitrogen is assumed (see Table 2). The values of γ and d for CO and CO_2 are taken from ref. [13].

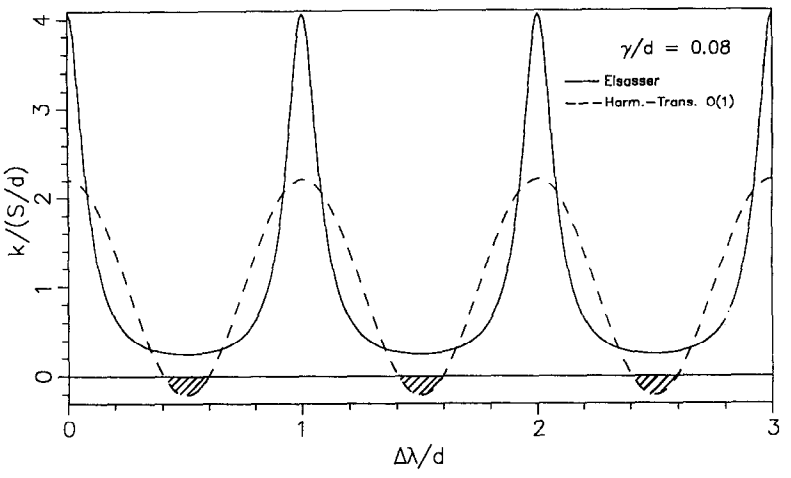

FIG. 6. Negative value of k_λ for $\gamma/d = 0.08$ and first-order approximation.

Table 1. Region of validity of the different orders of approximation

	γ/d
First order	$\geqslant \ln 2/2\pi = 0.110$
Second order	$\geqslant \ln \frac{8}{3}/4\pi = 0.0781$
Third order	$\geqslant 0.0691$

Table 2. Regions of total pressure for application of the HTM

	p_{total} [atm] at 300 K	
	CO at 4651 μm	CO_2 at 4255 μm
First order	≥6.7	≥0.55
Second order	≥4.7	≥0.39
Third order	≥4.2	≥0.34

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MODELE HARMONIQUE DE TRANSMISSION: APPROCHE NOUVELLE DU CALCUL DE TRANSFERT RADIATIF DANS LES GAZ EN TENANT COMPTE DE L'ELARGISSEMENT PAR PRESSION

Résumé—Une nouvelle méthode, le modèle harmonique de transmission (HTM), a été développée en tenant compte de l'effet de l'absorption gazeuse à structure linéaire dans les calculs de transfert radiatif multidimensionnel. La méthode est dérivée d'un développement du coefficient d'absorption et de l'intensité spectrale en série de Fourier et d'une adaptation par moindres carrés. Pour le cas monodimensionnel, une solution analytique est obtenue de la transmittivité spectrale moyenne qui peut être comparée directement aux modèles de bandes. Pour le cas multidimensionnel, on présente un système d'équations différentielles linéaires qui est de la même forme mathématique que l'équation du transfert radiatif. Par suite, toute méthode numérique convenable pour la résolution de l'équation du transfert radiatif peut être aussi employée pour résoudre l'équation différentielle résultant de la HTM.

DAS HARMONISCHE TRANSMISSIONSMODELL: EIN NEUES VERFAHREN FÜR DEN MEHRDIMENSIONALEN STRAHLUNGSAUSTAUSCH IN GASEN

Zusammenfassung—Es wurde eine neue Methode, das Harmonische Transmissionsmodell (HTM), entwickelt, das es ermöglicht, bei der mehrdimensionalen Berechnung des Strahlungsaustausches die Linienstruktur im Absorptionsverhalten von Gasen zu berücksichtigen. Die Herleitung der Methode basiert auf einer Entwicklung sowohl des Absorptionskoeffizienten als auch der spektralen Intensität in Fourier-Reihen und einer anschließenden Approximation der Strahlungstransportgleichung im Sinne kleinster Fehlerquadrate. Für eindimensionale Probleme ergibt sich eine analytische Lösung für die mittlere Transmission, die direkt den Bandmodellen gegenübergestellt werden kann. Bei mehrdimensionalen Problemen wird ein System von Differentialgleichungen angegeben, dessen Gleichungen dieselbe mathematische Struktur aufweisen wie die Strahlungstransportgleichung. Somit kann jede numerische Methode, die zur Lösung der Strahlungstransportgleichung geeignet ist, auch zur Lösung des Differentialgleichungssystems, das sich aus dem Harmonischen Transmissionsmodell ergibt, eingesetzt werden.

ГАРМОНИЧЕСКАЯ МОДЕЛЬ: НОВЫЙ ПОДХОД К РАСЧЕТУ МНОГОМЕРНОГО РАДИАЦИОННОГО ТЕПЛОПЕРЕНОСА В ГАЗАХ ПРИ УЧЕТЕ УШИРЕНИЯ ПОД ДЕЙСТВИЕМ ДАВЛЕНИЯ

Аннотация—Разработан новый метод, а именно, гармоническая модель пропускания, объясняющая эффект линейчатой структуры поглощения в газе при расчетах многомерного радиационного теплопереноса. Предложенный метод заключается в разложении коэффициента поглощения и спектральной интенсивности в ряды Фурье и использовании в уравнении радиационного переноса метода наименьших квадратов. В одномерном случае получено аналитическое решение для среднего по спектру коэффициента пропускания, которое можно непосредственно сравнить с результатами зонных моделей. Для многомерного случая представлена система дифференциальных уравнений, имеющих такую же математическую форму, как и уравнение радиационного переноса, Следовательно, для решения дифференциального уравнения, полученного на основе гармонической модели, может использоваться любой численный метод, годный для решения уравнения радиационного переноса.